Geotechnical Safety and Risk IV – Zhang et al. (eds)
© 2014 Taylor & Francis Group, London, ISBN 978-1-138-00163-3

Effect of slenderness ratio on the reliability-based serviceability limit state design of augered cast-in-place piles

S.C. Reddy & A.W. Stuedlein
Oregon State University, Corvallis, Oregon, USA

ABSTRACT: This study investigated factors that control the reliability of Augered Cast-In-Place (ACIP) piles in predominately cohesionless soils under axial compression at the Serviceability Limit State (SLS). A simple probabilistic hyperbolic model was used to account the uncertainty in the load-displacement relationship using correlated bivariate curve-fitting parameters. Contrary to previous studies, the curve-fitting parameters were found to be dependent on pile slenderness ratio (D/B) and the effect of D/B and other pertinent variables (e.g., uncertainty in capacity, displacement) on SLS reliability was investigated using a First-Order Reliability Method (FORM). The D/B ratio had a considerable effect on foundation reliability, illustrating the importance of the dependence between the load-displacement behavior (i.e. curve-fitting parameters) and pile geometry and stiffness. In general, the uncertainty in the capacity model had a larger effect on reliability than that of the allowable displacement; the reliability index was found to approach an upper bound limit regardless of the level of uncertainty in allowable displacement and the pile capacity model.

1 INTRODUCTION

The behavior of geotechnical systems under loading is often difficult to predict due to the inherent heterogeneity of the geologic environment. Because the compositional and mechanical properties of soils are variable, many parameters used in geotechnical design are uncertain. Traditionally, the uncertainty associated with many geotechnical design parameters has been assessed jointly using a deterministic global factor of safety; which is frequently based on engineering judgment and experience. Reliability-Based Design (RBD) procedures can overcome many of the restrictions of traditional design checks (e.g. Allowable Stress Design [ASD]), and explicitly incorporate the uncertainty in the individual variables and their potential correlation into the overall model. The probability of failure for a prescribed limit state that results thus allows a quantitative assessment of risk. As a result, RBD is quickly becoming the preferred alternative as the demand for risk management in geotechnical engineering continues to grow.

Modern RBD codes, in which partial safety factors are calibrated with respect to a specific limit state (e.g. Ultimate Limit State [ULS], Serviceability Limit State [SLS]), are now mandated for design of bridge foundation elements (e.g. American Association of State Highway and Transportation Officials [AASHTO] Load Resistance Factor Design [LRFD]). RBD procedures for Augered Cast-In-Place (ACIP) piles (e.g., Stuedlein et al. 2012) are not yet accepted in codes.

Owing to the lack of model statistics for pile displacement, foundation reliability at the SLS is not as well understood compared to the ULS (Phoon et al. 2006). In order to assess foundation reliability at the SLS, Phoon (2006) proposed a simple probabilistic hyperbolic model that accounts for the uncertainty in the entire load-displacement relationship using a bivariate random vector consisting of hyperbolic curve-fitting parameters, which were found to be correlated and non-normally distributed. Phoon & Kulhawy (2008) describe a translational model to incorporate the correlated random variables into reliability calculations using a database of 40 loading tests on ACIP piles.

This study used an expanded database to investigate factors affecting the reliability of ACIP piles at the SLS. Contrary to Phoon & Kulhawy (2008), the hyperbolic model parameters were determined to be dependent on the pile slenderness ratio. The dependence was removed by transforming the model parameters, which were then used to assess foundation reliability for different pile geometries. In order to determine the variables which govern reliability, a parametric study was conducted by varying the mean and uncertainty of allowable displacement, uncertainty of predicted resistance, and the slenderness ratio.

2 PROBABILISTIC HYPERBOLIC MODEL AT THE SLS

To ensure that a specified level of performance of a structure is met, it is necessary to assess the likelihood of failure at both the ULS and SLS using a consistent methodology. This study focuses on reliability at the SLS, defined by one or more predefined displacements that correspond to target allowable loads.

The load-displacement behavior of ACIP piles is influenced by multiple sources of uncertainty that can be implicitly accounted for by fitting load-displacement models to date from a load test database. In this approach, the aleatory and epistemic uncertainties resulting from the uniqueness of each load test and the error associated with measurements taken during testing are combined together and statistically characterized. Although a variety of functions can be used to model the load-displacement relationship, a hyperbolic curve was chosen herein in order to remain consistent with the work pioneered by Phoon (2006). The hyperbolic curve is represented using the applied load, Q, normalized by the slope-tangent capacity, Q_{STC} (Phoon et al. 2006):

$$\frac{Q}{Q_{STC}} = \frac{y}{k_1 + k_2 y} \tag{1}$$

where y = pile head displacement, and k_1 and k_2 are fitted coefficients. The reciprocal of k_1 and k_2 is equal to the initial slope and asymptotic (or ultimate) resistance, respectively. Model parameters from the new data were calculated using ordinary least squares regression, whereas the parameters in the Chen (1998) and Kulhawy & Chen (2005) database were obtained directly from Phoon & Kulhawy (2008).

3 DATABASE

The expanded database included 87 load tests on ACIP piles constructed in predominately cohesionless soils. Forty loading tests were collected by Chen (1998) and Kulhawy & Chen (2005), 23 were compiled by McCarthy (2008), ten were reported by Stuedlein et al. (2012), ten were collected by Park et al. (2012), three were reported by Mandolini et al. (2002), and one loading test was selected from O'Neill et al. (1999). Table 1 shows the range of pile embedment depth, D, diameter, B, slenderness ratio, D/B, average SPT-N along the pile shaft, N_{avg}, and Q_{STC}.

Table 1. Range of variables observed in the database.

Variable	D (m)	B (mm)	D/B	N_{avg} (bl/0.3 m)	Q_{STC} (kN)
Minimum	7.5	300	20.0	4	367
Maximum	29.0	800	68.5	54	5300

4 RANDOMNESS OF THE HYPERBOLIC MODEL PARAMETERS

In order for foundation reliability assessments to be unbiased, k_1 and k_2 must be statistically independent from other deterministic variables in the database (e.g., SPT-N and D/B). Based on the Kendall's tau test (Daniel 1990) and adopting a 5 percent significance level ($\alpha = 5$), k_1 and k_2 are independent of average SPT-N with p-values = 0.81 and 0.93, respectively. However, convincing evidence (p-values < 0.05) suggested that both k_1 and k_2 were dependent on D/B. Figure 1 shows

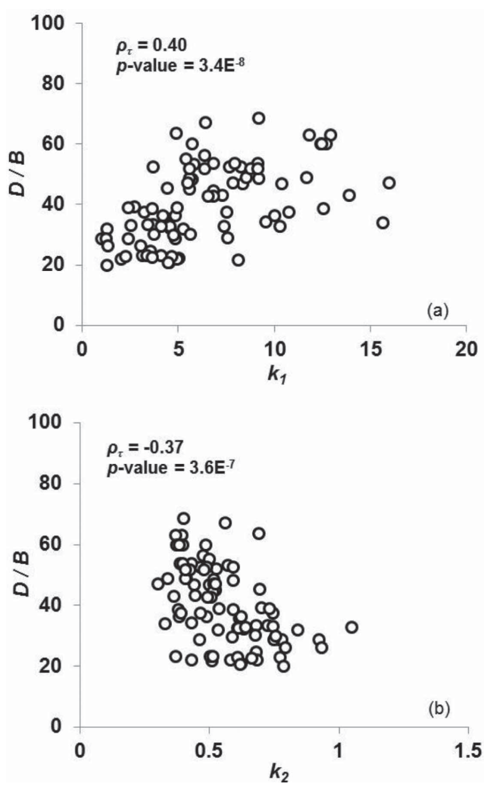

Figure 1. The dependence between slenderness ratio, D/B, and model parameters, (a) k_1 and (b) k_2 and the corresponding Kendall tau correlation coefficients and p-values.

moderately strong dependence between k_1, k_2, and D/B and the corresponding Kendall's tau correlation coefficient, ρ_τ, and p-value.

It is worthwhile to note that these correlations make physical sense in that a smaller k_1 represents a stiffer pile which corresponds to a smaller slenderness ratio; whereas a smaller k_2 indicates a larger ultimate resistance which likely relates to a larger D and D/B because of the narrow range of B in the database.

5 TRANSFORMATION OF THE MODEL PARAMETERS

In order to accurately model the uncertainty in the load-displacement relationship for the assessment of foundation reliability at the SLS, the correlation between k_1 and k_2 must be considered (Phoon & Kulhawy 2008). Figure 2a shows the inverse correlation between k_1 and k_2, and the corresponding ρ_τ and p-value, where a large (small) k_1 and small (large) k_2 indicates a slowly (quickly) decaying function and a less (more) well-defined and larger (smaller) asymptote.

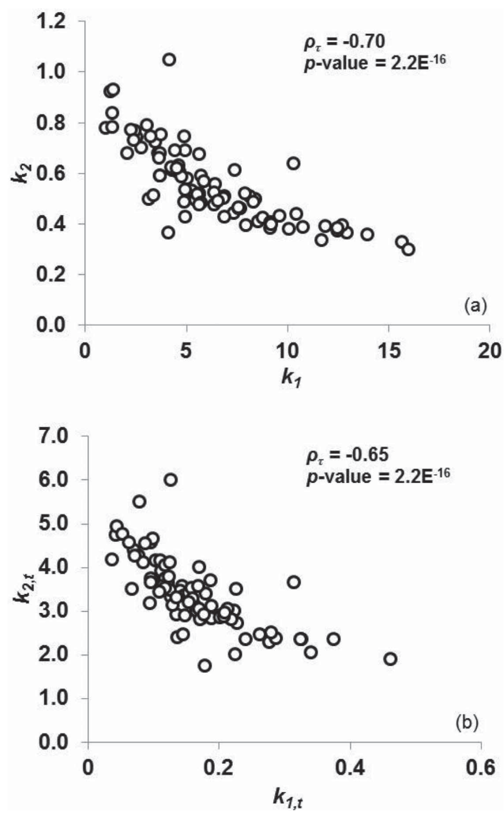

Figure 2. Correlation between model parameters (a) k_1 and k_2 and (b) $k_{1,t}$ and $k_{1,t}$ and the corresponding Kendall tau correlation coefficients and p-values.

In order to perform unbiased reliability analyses at the SLS, the correlation between model parameters and D/B must be considered. The dependence of k_1 and k_2 on D/B was removed by transforming the model parameters to:

$$k_{1,t} = k_1 \frac{B}{D} \tag{2a}$$

$$k_{2,t} = k_2 \sqrt{\frac{D}{B}} \tag{2b}$$

After transforming k_1 and k_2 to $k_{1,t}$ and $k_{2,t}$, the Kendall's tau correlation test between $k_{1,t}$ and $k_{2,t}$, and D/B produced p-values = 0.78, 0.56, indicating no correlation. Similarly, the model parameters remained independent of SPT-N following transformation. Figure 2b illustrates the correlation between $k_{1,t}$ and $k_{2,t}$ is slightly reduced but remains valid after transformation.

To assess foundation reliability using the translational model approach described in Phoon & Kulhawy (2008), the marginal distributions of $k_{1,t}$ and $k_{2,t}$ must be determined. Figure 3a and b shows the empirical, fitted normal, and fitted lognormal marginal Cumulative Distribution Functions (CDF) of $k_{1,t}$ and $k_{2,t}$, respectively. Also shown is the sample mean, $\bar{k}_{i,t}$, standard deviation, $\sigma_{i,t}$, and $COV_{i,t}$, defined as the standard deviation divided by the mean, of the model parameters.

Based on the Anderson-Darling goodness-of-fit test (Anderson & Darling 1952) and $\alpha = 5$ percent, there was no evidence to reject the null hypothesis of lognormality for $k_{1,t}$ and $k_{2,t}$. Therefore, $k_{1,t}$ and

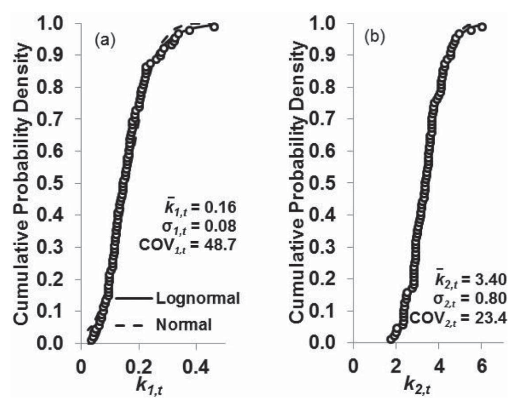

Figure 3. Empirical, lognormal, and normal marginal cumulative distributions for the transformed hyperbolic model parameters: (a) $k_{1,t}$, and (b) $k_{2,t}$.

$k_{2,t}$ were assumed to follow a lognormal distribution for the purpose of assessing foundation reliability at the SLS.

6 TRANSLATIONAL MODEL FOR BIVARIATE PROBABILITY DISTRIBUTIONS

The translational model approach for describing the marginal distributions of $k_{1,t}$ and $k_{2,t}$ requires the use uncorrelated standard normal random variables Z_1 and Z_2 (mean = 0, standard deviation = 1).

This study followed the basic procedure outlined in Phoon & Kulhawy (2008). First, Z_1 and Z_2 are converted into correlated random variables X_1 and X_2 (Phoon & Kulhawy 2008):

$$X_1 = Z_1 \tag{3a}$$

$$X_2 = Z_1\rho_{\ln} + Z_2\sqrt{1-\rho_{\ln}^2} \tag{3b}$$

where $\rho_{\ln}$ is an equivalent-normal correlation coefficient:

$$\rho_{\ln} = \frac{\ln\left[\rho\sqrt{\left(e^{\zeta_{1,t}^2}-1\right)\left(e^{\zeta_{2,t}^2}-1\right)}+1\right]}{\zeta_{1,t}\zeta_{2,t}} \tag{4}$$

and where $\lambda_{1,t}$, $\zeta_{1,t}$ and $\lambda_{2,t}$, $\zeta_{2,t}$ are the approximate lognormal mean and standard deviation of $k_{1,t}$ and $k_{2,t}$, respectively, and ρ is the standard product-normal correlation coefficient for two normally distributed variables. The second moment statistics in Equation 4 were calculated as:

$$\zeta_{i,t} = \sqrt{\ln\left(1+\sigma_{i,t}^2/\bar{k}_{i,t}^2\right)} \tag{5a}$$

$$\lambda_{i,t} = \ln\left(\bar{k}_{i,t}\right) - 0.5\zeta_{i,t}^2 \tag{5b}$$

The correlated, lognormal marginal distributions of $k_{1,t}$ and $k_{2,t}$ were thus simulated using:

$$k_{1,t} = e^{\left(\zeta_{1,t}X_1+\lambda_{1,t}\right)} \tag{6a}$$

$$k_{2,t} = e^{\left(\zeta_{2,t}X_2+\lambda_{2,t}\right)} \tag{6b}$$

In order to adequately reproduce the uncertainty in the observed load-displacement curves, $k_{1,t}$ and $k_{2,t}$ must be back-transformed into k_1 and k_2. This study calculated k_1 and k_2 using deterministic values of D/B because the uncertainty associated with pile geometry could not be evaluated from the database.

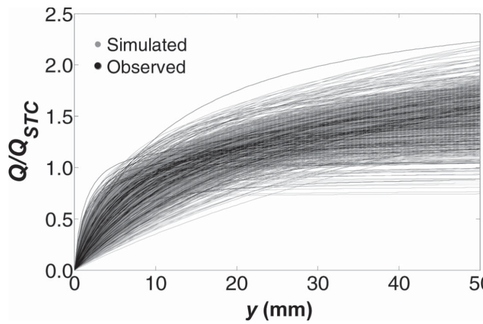

Figure 4. Observed and simulated load-displacement curves using the translational model.

Figure 4 shows the fitted load-displacement curves based on the observed loading tests and those generated using the procedure for simulating $k_{1,t}$ and $k_{2,t}$ outlined above. A sufficient number of deterministic values of D/B were used to back-transform $k_{1,t}$ and $k_{2,t}$ into k_1 and k_2 in order to adequately capture the uncertainty present in the observed load-displacement curves. In general, the observed and simulated load-displacement curves are in good agreement, and the translational model can be confidently used to assess foundation reliability at the SLS using the database herein.

7 RBD FOR THE SERVICEABILITY LIMIT STATE USING A FIRST-ORDER RELIABILITY METHOD

The SLS is reached when foundation displacement, y, equals or exceeds allowable settlement, y_a. This study followed the approach outlined in Phoon & Kulhawy (2008), where the SLS can be evaluated using a performance function, P:

$$P = y_a - y(Q) \tag{7}$$

Failure is defined as $P \le 0$, and the probability of exceeding the SLS, p_f, is:

$$p_f = \Pr(P \le 0) \tag{8}$$

By combining Equations 1, 7, and 8, and defining a deterministic mean global factor of safety, FS, the probability of failure is:

$$p_f = \Pr\left(\frac{y_a}{k_1+k_2 y_a} < \frac{1}{FS}\frac{Q'}{Q'_p}\right) \tag{9}$$

where Q' and Q'_p correspond to the applied load and predicted pile capacity, respectively. In order estimate foundation reliability at the SLS, the reliability index, β, defined as the number of standard deviations between the mean of the multivariate resistance distribution and the limit state surface, was calculated as:

$$\beta = -\Phi^{-1}\left(p_f\right) \qquad (10)$$

where Φ^{-1} is the inverse standard normal function.

A First-Order Reliability Method (FORM) was used to estimate foundation reliability at the SLS. First, each random variable in the limit state function (k_1, k_2, y_a, Q', Q'_p) was transformed into a standard normal variable, such that the difference in magnitude of the random variables was eliminated (Hasofer & Lind 1974). Then the probability of failure was estimated by considering the area beneath the multivariate distribution where $P \leq 0$. The FORM approach assumes that the limit state function is linear at the failure point, and therefore may not be appropriate for situations where p_f is large. However, this approach is considered sufficient for most geotechnical applications where the target probabilities of failure are very small.

8 FACTORS AFFECTING FOUNDATION RELIABILITY AT THE SLS

This study assessed the factors which govern foundation reliability at the SLS by calculating multiple reliability indices using FORM. Each variable in Equation 9 was assumed to follow a lognormal distribution, whereas the second moment statistics for $k_{1,t}$ and $k_{2,t}$ were obtained directly from the database. The mean and COV of allowable displacement was varied from 10 to 50 mm and 5 to 85 percent, respectively. The applied load and predicted pile capacity were assumed to be unit mean variables, where COV(Q') = 20 percent based on recommendations from Paikowsky et al. (2004). The COV of the predicted pile capacity was varied from 5 to 85 percent, corresponding to different capacity prediction methods with varying degrees of uncertainty. A $FS = 3$ was selected based on that commonly adopted in current practice (Phoon & Kulhawy 2008). Slenderness ratios of 25 and 65 were selected in order to cover the range of D/B values in the database and illustrate the effect of pile geometry on β.

Figures 5a-e illustrate the effect of changing the mean y_a, COV(y_a), COV(Q'_p), and D/B on foundation reliability. Foundation reliability decreases more rapidly for increasing uncertainty in Q'_p when COV(y_a) and COV(Q'_p) are relatively small (5–45 percent) and $y_a > 20$ mm. In general, COV(Q'_p) has a larger effect on β as compared to COV(y_a),

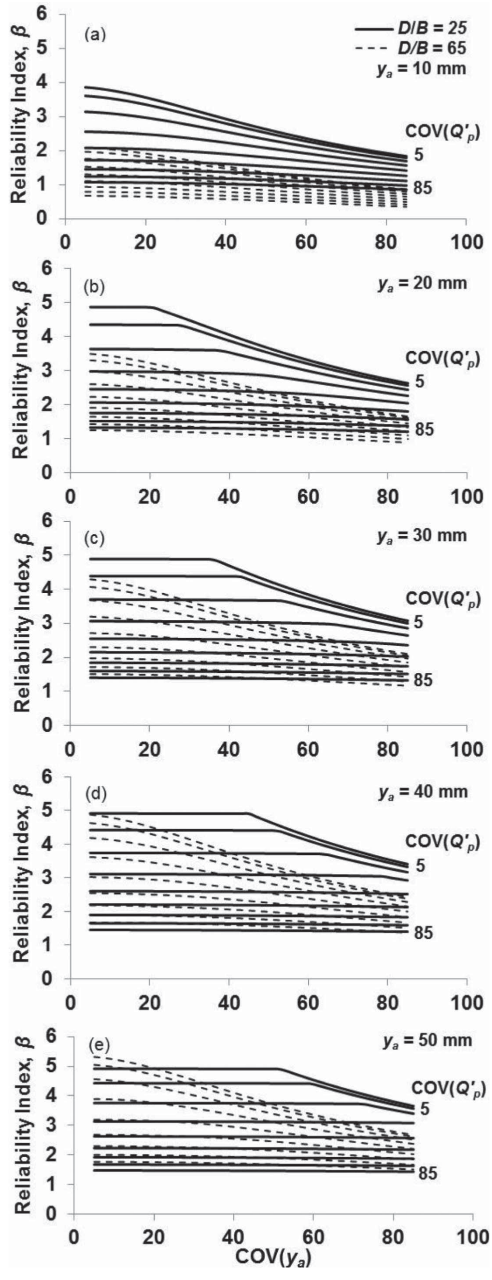

Figure 5. The effect of COV(y_a), COV(Q'_p), and D/B on β for mean y_a equal to (a) 10 mm, (b) 20 mm, (c) 30 mm, (d) 40 mm, and (e) 50 mm using $k_{1,t}$ and $k_{2,t}$ developed herein.

regardless of the level of uncertainty in Q'_p and y_a. The same general trend was observed at different levels of mean allowable displacement, where the change in β was more prominent for larger y_a. Overall, β was larger for larger mean allowable displacements if all other variables in the performance function remained constant. At large allowable displacements (i.e. $y_a = 50$ mm), β was observed to be largely insensitive to the level of uncertainty in y_a, compared to Q'_p. This illustrates the advantage of an accurate ACIP design methodology. At large allowable displacements, β approaches an upper bound limit for each level of COV(Q'_p) as COV(y_a) decreases. For a mean $y_a < 40$ mm, β is smaller for larger D/B, whereas the opposite is true for $y_a \geq 50$ as shown in Figure 5e. Thus, accounting for the correlation between the hyperbolic model parameters and D/B is critical when estimating the reliability of ACIP piles at the SLS.

In order to illustrate the effect of slenderness ratio on foundation reliability at the SLS, reliability indices calculated herein may be compared to those reported in Phoon & Kulhawy (2008). Using the statistics for y_a and Q'_p recommended by a Phoon and Kulhawy (2008) and a $D/B = 25$, $\beta = 2.214$ ($p_f = 1.34\%$) is computed, which is in good agreement with the previously reported value (2.210). However, for longer piles, say with a $D/B = 65$, the reliability index, β, equals 1.774 ($p_f = 3.80\%$), a significantly different value than previously computed.

9 SUMMARY AND CONCLUSIONS

This paper investigated the effect of varying model statistics in reliability-based serviceability limit state design of ACIP piles installed in predominately cohesionless soils. First, a database consisting of load tests conducted on ACIP piles in cohesionless soils was compiled, and the uncertainty in the entire load-displacement relationship was reduced to a correlated bivariate vector containing the hyperbolic model parameters. Contrary to Phoon & Kulhawy (2008), both model parameters were found to be correlated to pile slenderness ratio. Subsequent analyses used transformed model parameters to avoid the undesirable effect of parameter dependence on geometric variables.

The effect of varying the mean and uncertainty of the allowable displacement and the uncertainty of the capacity prediction method on the computed reliability index was assessed. In general, changing the uncertainty in Q'_p had a larger effect on β compared to y_a. Overall, β was larger for larger mean allowable displacements when all other variables in the performance function were unchanged. At larger allowable displacements, β was found to approach an upper bound limit and shown to be largely insensitive to the level of uncertainty in y_a, compared to Q'_p. Because of the dependence of the model parameters on pile stiffness and geometry, β was found to be sensitive to D/B, and illustrates the importance of accounting for this correlation in RBD.

REFERENCES

Anderson, T.W. & Darling, D.A. 1952. Asymptotic theory of certain goodness-of-fit criteria based on stochastic processes. *The Annals of Mathematical Statistics* 23(2): 193–212.

Chen, J.R. 1998. Case history evaluation of axial behavior of augered cast-in-place piles and pressure-injected footings. M.S. Thesis. Cornell University. Ithaca, New York.

Daniel, W.W. 1990. Applied nonparametric statistics. Boston: PWS-Kent.

Hasofer, A.M. & Lind, N. 1974. An exact and invariant first-order reliability format. *Journal of Engineering Mechanics* 100(1): 111–121.

Kulhawy, F.H. & Chen, J.R. 2005. Axial compression behavior of augered cast-in-place (ACIP) piles in cohesionless soils. In C. Vipulanandan & F.C. Townsend (eds), *Advances in Deep Foundations; Proc. of GeoFrontiers Congress (GSP 132), Austin, Texas, 24–26 January 2005.* Reston: ASCE.

Mandolini, A., Ramondini, M., Russo, G. & Viggiani, C. 2002. Full scale loading tests on instrumented CFA piles. In M.W. O'Neill & F.C. Townsend (eds), *An International Perspective on Theory, Design, Construction, and Performance; Proc. of Deep Foundations (GSP 116), Orlando, Florida, 14–16 February 2002.* Reston: ASCE.

McCarthy, D.J. 2008. Empirical relationships between load test data and predicted compression capacity of augered cast-in-place piles in predominately cohesionless soils. M.S. Thesis. University of Central Florida. Orlando, Florida.

O'Neill, M.W., Vipulanandan, C., Ata, A. & Tan, F. 1999. Axial performance of continuous-flight-auger piles for bearing. *Texas Dept. of Transportation report no. 7-3940-2*, Center for Innovative Grouting Materials and Technology: University of Houston.

Park, S., Roberts, L.A. & Misra, A. 2012. Design methodology for axially loaded auger cast-in-place and drilled displacement piles. *Journal of Geotechnical and Geoenvironmental Engineering* 138(12): 1431–1441.

Paikowsky, S.G. with contributions from Birgisson, B., McVay, M., Nguyen, T., Kuo, C., Baecher, G., Ayyab, B., Stenersen, K., O'Malley, K., Chernauskas, L. & O'Neill, M.W. 2004. Load and resistance factor design (LRFD) for deep foundations. *NCHRP report no. 507,* Transportation Research Board: Washington, DC.

Phoon, K.K. 2006. Serviceability limit state reliability-based design. In M.L. Lin, C.T. Chin, H.D. Lin, Y. Honjo, K.K. Phoon (eds), *New Generation Design Codes for Geotechnical Engineering Practice; Proc. of Int. Symp., Taipei, Taiwan 2–3 November 2006.* Singapore: World Scientific Publishing Company.

Phoon, K.K., Chen, J.R. & Kulhawy, F.H. 2006. Characterization of model uncertainties for auger cast-in-place (ACIP) piles under axial compression. In R.L. Parsons, L.M. Zhang, W.D. Guo, K.K. Phoon, & M. Yang (eds), *Foundation Analysis and Design—Innovative Methods (GSP 153).* Reston: ASCE.

Phoon, K.K. & Kulhawy, F.H. 2008. *Reliability-based design in geotechnical engineering.* New York: Taylor & Francis.

Stuedlein, A.W., Neely, W.J. & Gurtowski, T.M. 2012. Reliability-based design of augered cast-in-place piles in granular soils. *Journal of Geotechnical and Geoenvironmental Engineering* 138(6): 709–717.